Beam Dynamics of Chirp Scheme in Storage Rings

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Beam Dynamics Mini-Workshop on Deflecting/Crabbing Cavity
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Applications of deflecting cavities in SR

- Two major applications for deflecting cavities:
  - Restoring head-on collisions in crab crossing in colliders
    - Suppresses synchro-betatron resonances excited by crab crossing
  - Generating short X-ray pulses in light sources
    - Allows to take advantage of small vertical beam size to generate temporally short pulses
- Some beam dynamics issues are similar:
  - Additional impedance
  - Cavity generated beam noise
- Some are different
  - Beam-beam related effects in colliders
  - Coupling increase and related nonlinear dynamics complications in light sources
- Major difference is deflection plane: vertical for light sources and horizontal for colliders

Advanced Photon Source parameters

- Here we will discuss the beam dynamics from the light source point of view
- Simulations or estimations are done using APS parameters

Energy	7 GeV
Circumference	1104 m
Horizontal emittance	2.5 nm rad
Vertical emittance	40 pm rad
Deflecting voltage	2 MV
Deflecting frequency	2.8 MHz
β_y at cavity location	5 m

Effect on the beam

- Less than total kick cancellation at the second cavity could lead to beam emittance increase and to orbit distortion
- Nonlinear beam dynamics is affected
- Cavities introduce additional impedance, and therefore can affect single-bunch and multi-bunch instabilities

Effect on emittance

- In a real machine, many effects could lead to emittance degradation
  - Various errors and imperfections are first things coming to mind
- However, even in a perfect machine the emittance can increase many fold
  - Path length dependence on the particle energy leads to incomplete kick canceling in the second cavity
  - Betatron phase advance dependence on energy (chromaticity) leads to closed bump condition breaking
  - Sextupoles between cavities introduce nonlinearities that generate betatron phase advance dependence on amplitude and linear coupling between horizontal and vertical planes

Momentum compaction

- This effect comes from the path length difference between the cavities for particles with different energy
- This effect is present even if there are no errors and nonlinearities
- For a particle with energy deviation δ_i, the time of flight differential

$$\Delta t_i = \alpha_c \delta_i T_0$$

- Additional kick after the second cavity is

$$\Delta y_i' = \frac{-V \omega \Delta t_i}{E}$$

which gives emittance increase of

$$\frac{\Delta \epsilon_y}{\epsilon_y} = \frac{\sqrt{\sigma_{y'}^2 + \sigma_{\Delta y'}^2}}{\sigma_{y'}} - 1$$

- For APS case, it gives about 0.3% increase of emittance in a single turn which gives negligible effect on overall emittance increase

Chromaticity

- The second cavity is placed at $n\pi$ phase advance to cancel the kick of the first cavity
- If there is non-zero chromaticity ξ_y between the cavities, the phase advance of a particle with δ_i is changed by $-2\pi\xi_y\delta_i$ which leads to a particle position change at the second cavity

$$y_2 = \beta y'_1 \sin(2\pi\xi_y\delta_i)$$

- The rms value of the residual amplitude is

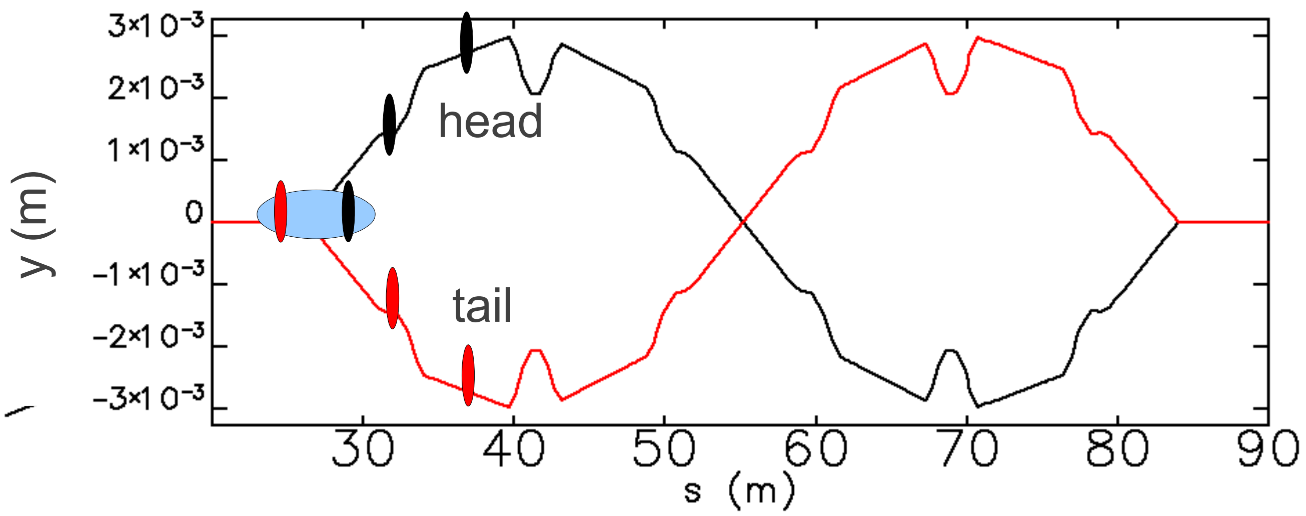
$$\sigma_{y_2} = 2\pi\xi_y\beta\frac{V\omega}{E}\sigma_\delta\sigma_t$$

- For APS parameters with uncompensated chromaticity, this works out to be over 50% of the nominal vertical beam size of 11 μm
- **To avoid this emittance increase, sextupoles are required between the cavities**

Sextupole nonlinearities

- Sextupoles can affect in two ways:

- By introducing amplitude-dependent focusing
  - for particles going off-axis the kick cancellation at the second cavity is not perfect
- By introducing transverse coupling
  - deflecting cavities generate large vertical trajectories in sextupoles
  - Vertical trajectory in sextupoles creates coupling between large horizontal and small vertical emittances



Beam dynamics simulation methods

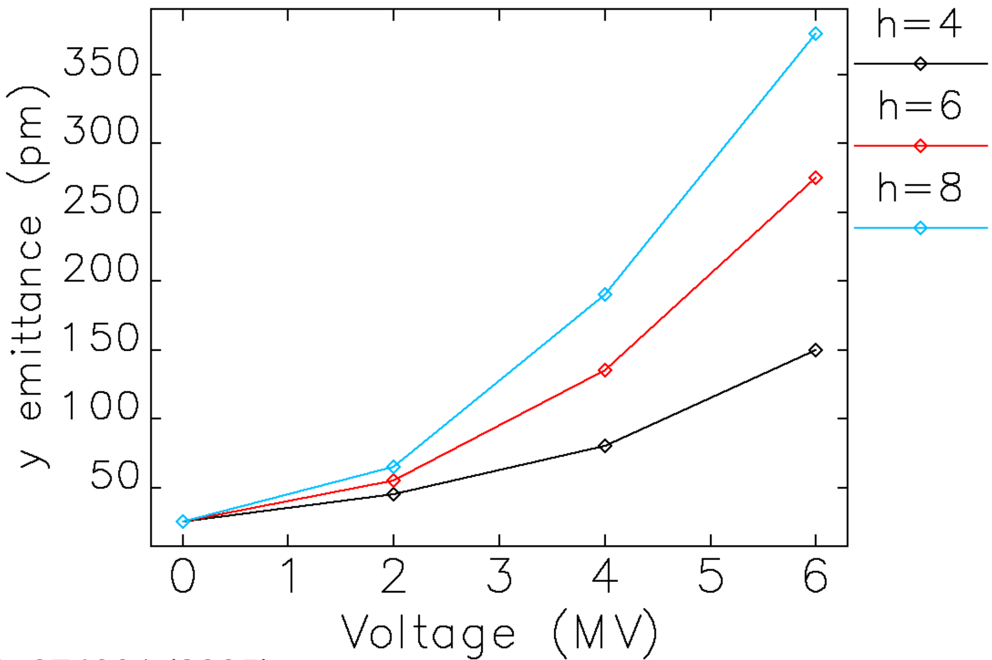
- We use tracking to simulate beam dynamics
- We use parallel elegant[1] typically utilizing 10-50 CPU cores
- Accelerating cavities are required to simulate synchrotron motion
- Synchrotron radiation is essential: to damp initial cavity effects
  - Tracking is done for 10k turns – about 4 damping times, to get beyond cavity "switch-on" effects
- Deflecting cavity is simulated as TM-like mode, deflection is radius independent resulting from combination of TM- and TE-like field[2]

[1]Y. Wang et al., AIP 877, 241 (2006).

[2]M. Nagl, tesla.desy.de/fla/publications/talks/seminar/FLA-seminar_230904.pdf

Initial results of the deflecting cavity application

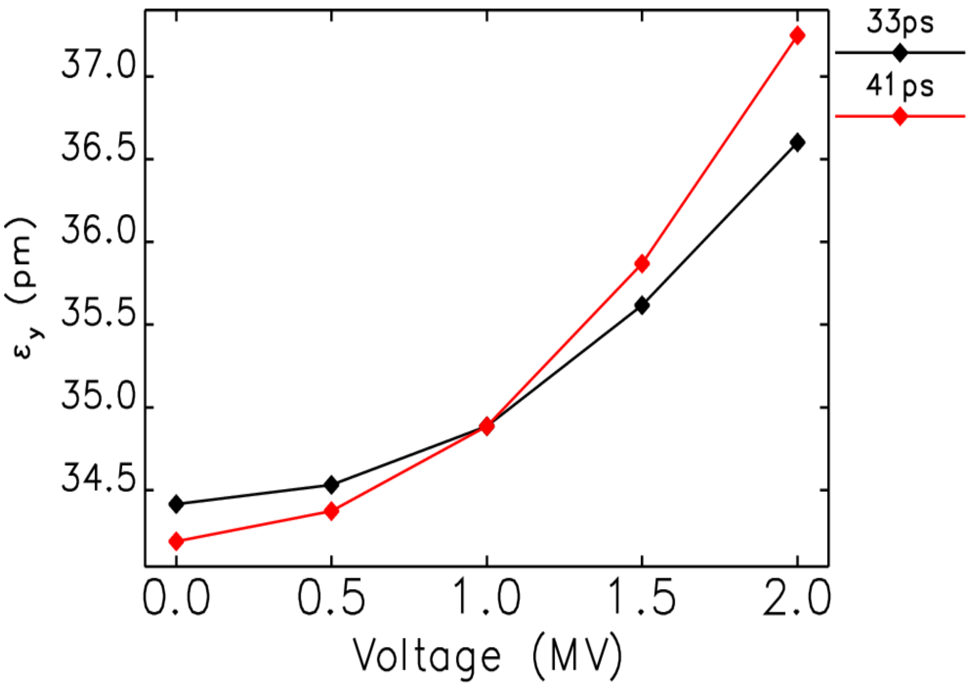
- Right away, we have found significant blow-up of vertical emittance due to increased coupling[1]
- We have found that main contribution in our case was coupling on vertical trajectories



[1]M. Borland, PRSTAB 8, 074001 (2005).

Vertical emittance after sextupole optimization

- Skew quadrupoles cannot be used to compensate for this source of coupling because it is longitudinally dependent
- Changing sextupole strength between cavities can help in reducing coupling
- We used single-pass tracking of a bunch of particle to minimize vertical emittance increase
- Vertical emittance growth below 10% was achieved
- Two bunch lengths corresponding to two different operating conditions are shown here



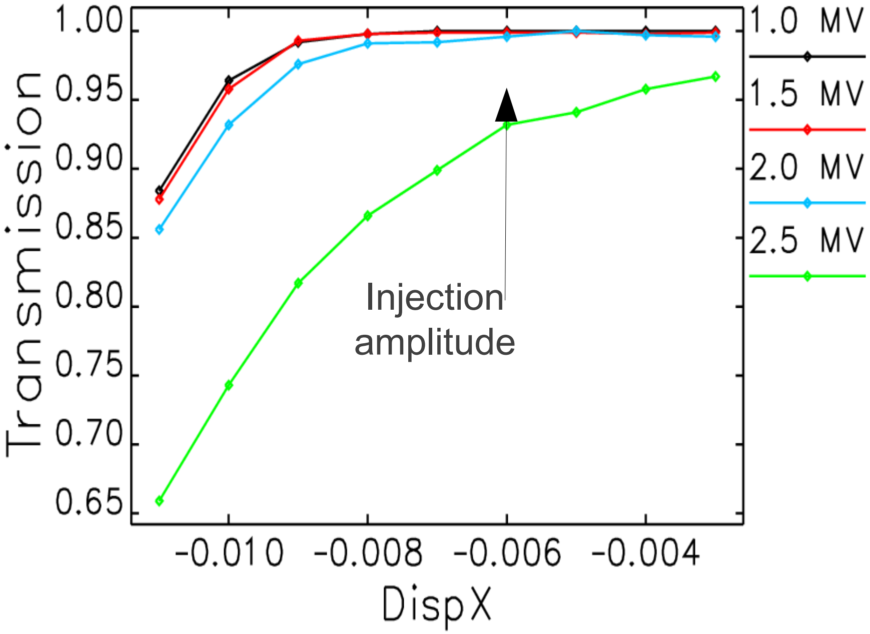
Nonlinear dynamics

- Light sources tend to minimize their beam emittance to the level where Dynamic Aperture (DA) and lifetime are barely enough for operation
- Many sextupole families are utilized to achieve workable DA and lifetime
- Sextupole optimization that was used to minimize vertical emittance blow-up changes local sextupole distribution and violates the sextupole optimization for nonlinear dynamics
- Even small reduction of DA and lifetime can be crucial
- It is important to simulate the deflecting cavity effects on nonlinear dynamics
- The cavity effects are defined by large vertical trajectories between deflecting cavities:
  - Physical acceptance is decreased
  - Additional linear and nonlinear coupling is introduced

Injection with deflecting cavities

- We simulated injection process as oscillations of the bunch with APS booster parameters at extraction (ε_x=100 nm, coupling 5%)
- Simulation is stragihtforward: ordinary particle tracking with deflecting cavities on

- We calculated particle transmission vs beam oscillation amplitude for different deflecting voltages
- Injection corresponds to 6 mm amplitude
- Limiting reason in our case is physical aperture

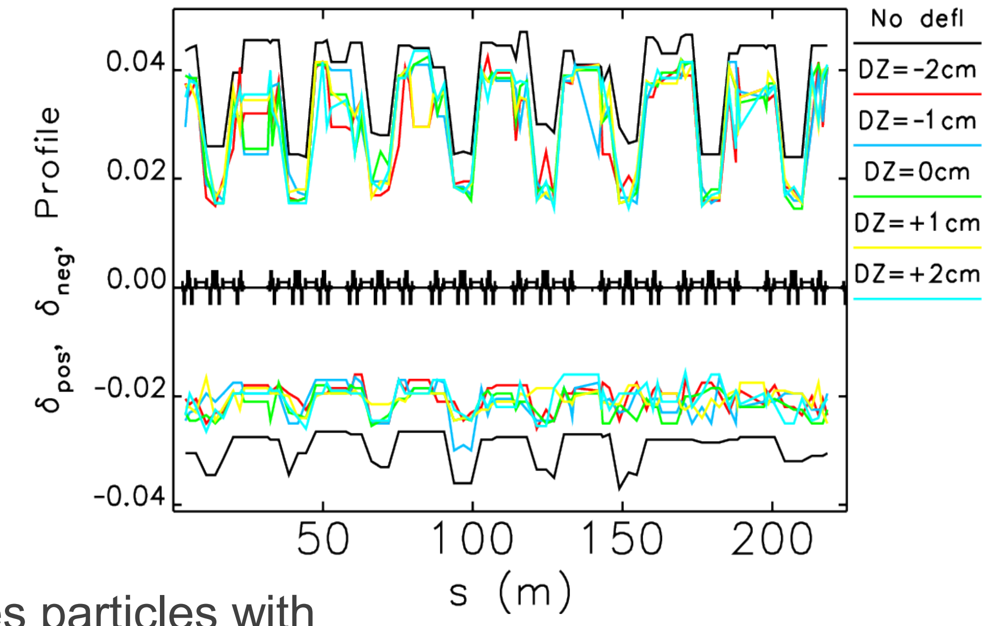


Lifetime with deflecting cavities

- Lifetime is calculated based on Local Momentum Aperture (LMA)[1]
- LMA is a single particle simulation, so its application is not as straightforward since different particles experience different deflecting kicks

We calculated LMA in the presence of deflecting cavities for particles with different longitudinal offsets. Reduction of LMA is significant.

The lifetime for different offsets agree to each other within 10%



It is because the simulation involves particles with large energy deviation which experience large synchrotron oscillations

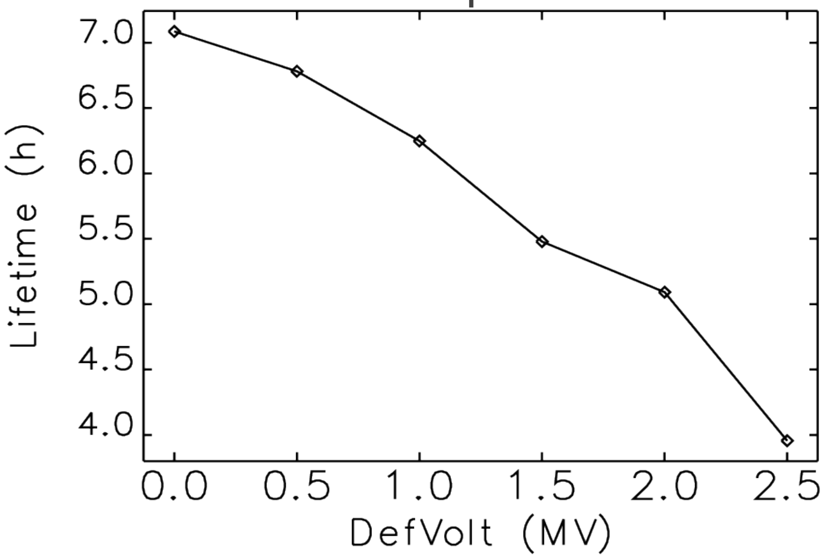
[1]A. Piwinski, DESY-98-179

Lifetime with deflecting cavities (2)

- We found significant lifetime reduction with deflecting voltage
- Unlike injection efficiency, increasing physical aperture didn't help
- Looking at the tracking results in detail, we have found that particles are lost on a nearby skew sextupole resonance which gets excited in the presence of increased coupling[1]

By optimizing sextupoles between cavities, we were able to completely recover the LMA and lifetime.

Lifetime reduction with original sextupoles



[1]M. Borland, AOP-TN-2011-021

Sextupole optimization

- Sextupoles between the cavities are needed to compensate for natural chromaticity
- At the same time large vertical trajectories in sextupoles lead to vertical emittance increase and nonlinear dynamics deterioration
- We showed that optimization of sextupoles between cavities allows to solve each problem separately
- Now we need to satisfy everything at the same time
- The best way to do it is to use multi-objective optimization, and do it as a part of overall lattice design

Sextupole optimization (2)

- The optimization is done using genetic optimizer
- Every optimizer evaluation consists of
  - Linear optics design (if required)
  - Interior sextupoles optimization for vertical emittance blowup minimization
  - Exterior sextupole optimization for DA/LMA
- The penalty functions are vertical emittance increase, DA area, and lifetime
- It is very CPU-hungry process, it requires parallel computations, but it gives satisfactory results
  - We are able to achieve satisfactory dynamic aperture and lifetime without any increase of vertical emittance
- **We have found that we can generate lattices that have acceptable emittance growth, lifetime, and DA at the same time**
- DA/LMA evaluation with cavities on is not included in optimization yet

Deflecting voltage tolerances

- The voltage could vary in amplitude and phase, and variations at both cavities could follow each other (common-mode) or not (differential-mode)
- Common-mode variations affect the beam only between the cavities
  - Important for colliders
  - Not as important for light sources because the beam size between cavities is greatly increased
- Differential-mode variations affect the beam everywhere
  - Give very tight tolerances for light sources due to small vertical beam sizes
- Will not talk about common-mode tolerances

Differential mode tolerances

- When the voltage waveform in the second cavity does not exactly follow the first cavity, the resulting effect of two cavities on the beam is non-zero:

$$V\sin(\omega t)-(V+\Delta V)\sin(\omega t+\Delta\phi)\approx V\cos(\omega t)\sin(\Delta\phi)-\Delta V\sin(\omega\phi)$$

- The first term provides a net orbit kick because its value is non-zero at the center of the bunch (t=0)

- The second term generates beam tilt outside of the deflecting cavities and affects projected beam sizes

- The effect can be treated as a single source orbit distortion and a single deflecting cavity with voltage ΔV.

Tolerances: Orbit

- Want to keep orbit variation under some fraction of nominal beam emittance (total APS beam motion budget in terms of beam motion invariant is 1% of beam emittance)
- For a single-source orbit distortion the orbit invariant is:

$$A=\gamma x^2+2\alpha x x'+\beta x'^2=\frac{\theta^2\beta_0}{4\sin^2(\pi\nu)}<0.01\epsilon_y$$

where θ is the orbit kick

$$\theta\approx\frac{V}{E}\sin\Delta\phi$$

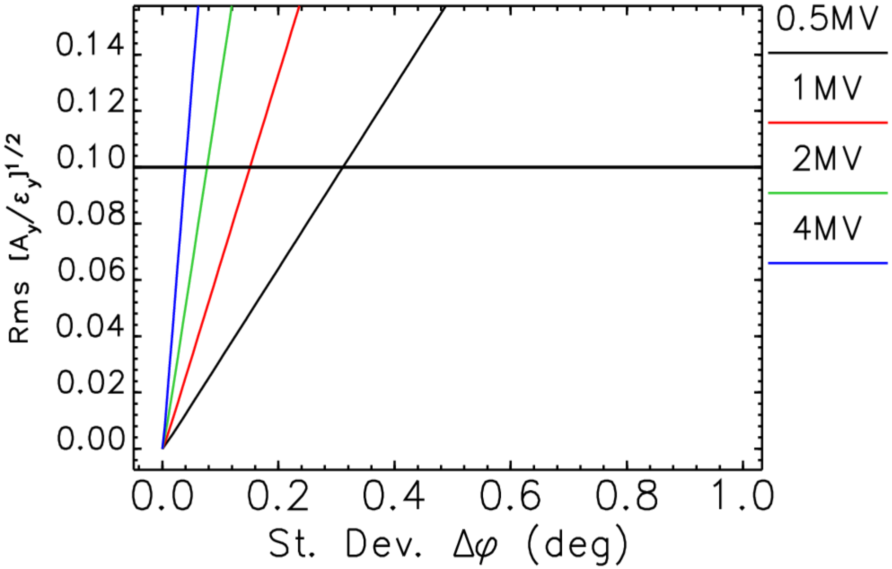
- For the phase tolerance, we get:

$$\Delta\phi<2\sin(\pi\nu)\sqrt{\frac{0.01\epsilon_y}{\beta_0}}\frac{E}{V}$$

- Using APS parameters, we get:

$\Delta\phi<$ 0.08 deg or 80 fs

Orbit motion vs phase error (simulations)



- Simulations are used to get exact tolerances

Tolerances: Emittance

- Various errors affect the outside beam sizes
  - Differential deflecting voltage
  - Vertical betatron phase advance not equal to $N*\pi$
  - Beta function mismatch
  - Cavity and magnet roll
- All these errors except differential deflecting voltage are static
  - Beta function error can be compensated by changing relative voltage of second cavity
  - Phase advance error can be compensated by changing relative voltage of first and second sets of cells in second cavity
  - Cavity roll is found to be a weak effect[1]
  - Magnet roll can be corrected with additional skew quadrupoles
- We will only look at effect of differential voltage errors

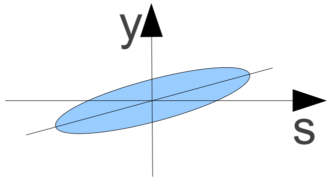
Tolerances: Emittance (2)

- A single cavity with voltage ΔV generates beam tilts around the ring:

$$\frac{dy(s)}{ds}=\frac{\Delta V}{E}\cdot\sin\left(\frac{\omega_{rf}s}{c}\right)\cdot\sqrt{\beta_0\beta}\cdot\frac{\cos(\phi-\pi\nu)}{2\sin(\pi\nu)}$$

$$\frac{dy'(s)}{ds}=\frac{\Delta V}{E}\cdot\sin\left(\frac{\omega_{rf}s}{c}\right)\cdot\sqrt{\frac{\beta_0}{\beta}}\cdot\frac{\sin(\phi-\pi\nu)-\alpha\cos(\phi-\pi\nu)}{2\sin(\pi\nu)}$$

Here *s* is coordinate inside the bunch:



- When bunch length σ_s is shorter than the deflecting rf wavelength, the resulting projected beam sizes are:

$$\sigma_{y\ defl\ beam}^{2}=\sigma_y^2+\left(\frac{dy}{ds}\cdot\sigma_s\right)^2 \qquad \sigma_{y'\ defl\ beam}^{2}=\sigma_{y'}^2+\left(\frac{dy'}{ds}\cdot\sigma_s\right)^2$$

- If we require that the resulting beam size does not exceed $(1+f)\sigma_y$ then

$$\Delta V\approx 2\frac{\sqrt{2f}}{\sigma_s}\sqrt{\frac{\epsilon_y}{\beta_0}}\frac{E\,c\sin(\pi\nu)}{\omega_{rf}}$$

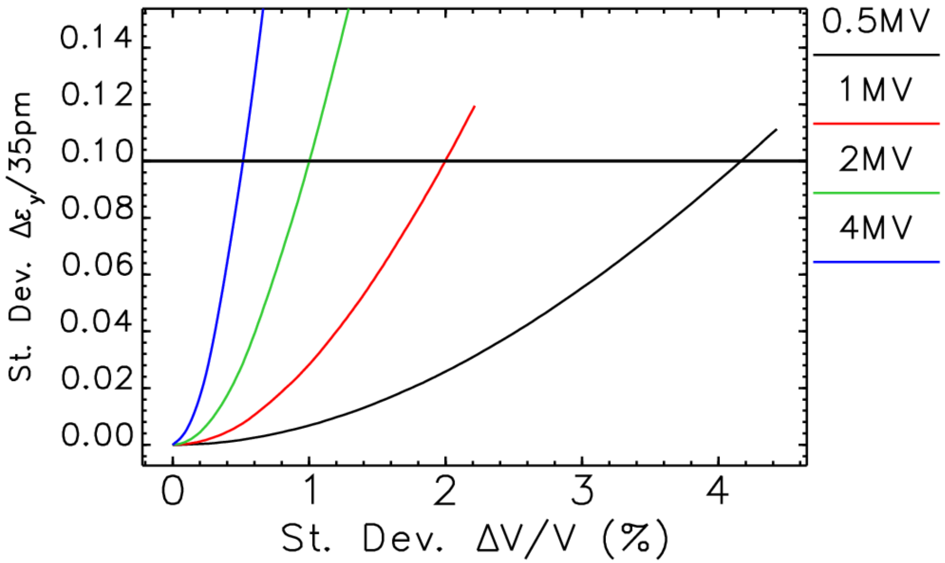
Used electron beam size here for simplicity. In reality, a convolution of electron and photon beam sizes matters here.

Tolerances: Emittance (3)

- If we require that the beam size increase does not exceed 10% of the total beam size, for APS parameters we get:

$$\frac{\Delta V}{V} < 0.01$$

- Realistic tracking simulations of the emittance sensitivity to the voltage errors show good agreement:



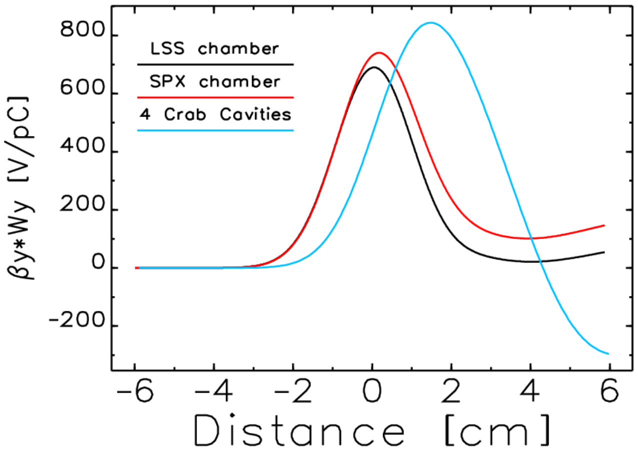
Collective effects

- Can be separated into short- and long-range effects
- Long-range effects generate multi-bunch instabilities
- Short-range wake fields limit single bunch current
- Since the exact knowledge of the HOM frequencies is not possible in advance, we first calculated the limiting resonator impedance for a single cavity assuming worst possible HOM frequency
- This was used as a guide in the cavity and HOM damper design
- We then used Monte-Carlo based simulations to confirm that the final cavity design provides instability growth slower than the synchrotron damping time

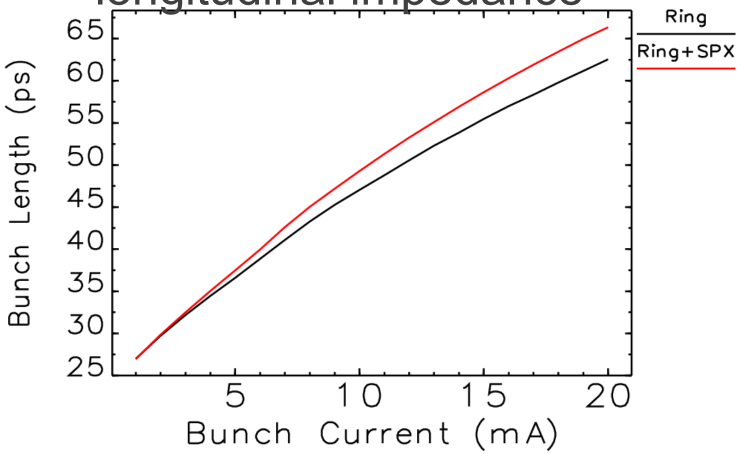
Collective effects (2)

- Short-range wake fields could limit single bunch current
- Additional impedance comes from cavities themselves and vacuum chamber transitions
- We found that cavities do not limit single bunch current due to additional bunch lengthening

Transverse wake field



Bunch lengthening due to longitudinal impedance



Conclusions

- Deflecting cavities could affect single particle beam dynamics through nonlinearities on large trajectories between the cavities
  - Sextupoles and nonlinearities of the deflecting fields could limit momentum and dynamics aperture
  - Sextupoles could greatly increase transverse coupling
- Could increase beam emittance and generate beam motion through rf noise in cavities
- Cavities introduce additional impedance, and therefore can affect single-bunch and multi-bunch instabilities